Tunable optical absorption in undoped graphene sandwiched between multilayer dielectric stacks with mirror symmetry

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Abstract. We theoretically investigate the optical absorption of an undoped graphene monolayer when put in a one-dimensional multilayer stack. Using the transfer matrix method, we perform numerical simulations and derive explicit analytical formulas for the optical absorption of the graphene monolayer at the center of the dielectric stack and find that the optical absorption uniquely depends on repetition number (r) and the unit layers structure. When sandwiched between unit layers structure composed of three dielectric materials (referred to as the "ABC" structure) with even values of r, the graphene monolayer absorbs 2.3% of visible to near-infrared light. This behavior is the same as if graphene were free-standing, not sandwiched between the dielectric stack. In contrast to that situation, in the ABC structure with odd values of r, also when the graphene monolayer is sandwiched between four materials (the "ABCD" structure) with any values of r, we can obtain optical absorption as large as 50% at particular refractive indices (n) of the constituent dielectric materials. The 50%-absorption is, in fact, the maximum optical absorption for any undoped monolayer material in the symmetric dielectric stacks. By varying r and n within the ABC or ABCD structures, we can finely adjust the optical absorption of graphene within the range of 0–50%, facilitating precise control for various optoelectronic applications.

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1. Introduction

In recent years, graphene and other two-dimensional (2D) materials have garnered significant attention due to their unique optical and electrical properties, holding promise for developments of novel nanodevices, especially in the fields of photonics and electronics [1–7]. However, while there is already noticeable interaction between light and matter in an atomically thin layer like graphene, larger optical absorption is indispensable [2, 3, 8–11]. The optical absorption of undoped graphene monolayer remains relatively constant across different wavelengths ($\pi\alpha \approx 2.3\%$), primarily originating from the interband transition [12–16], with $\sigma = e^2/4\hbar$ as the optical conductivity of undoped graphene is essential as a key factor determining the absorption [15]. Some reports have suggested the enhancement of optical absorption of graphene via doping [17–20], while the non-doping approach includes gain-assisted critical coupling [21], periodically patterning graphene into nanodisks [22, 23], harnessing the toroidal dipole-bound state in the continuum [24], and coupling it with hole-spaned photonic crystals [22, 25–27].

In 2018, Nulli *et al.* demonstrated that a mirrored multilayer structure, comprising repeated unit layers of two dielectric materials, each with a thickness of quarter-wavelength for each layer [28] could exponentially enhance the electric field of the incident light at the center of the structure. When graphene is placed at such a position, the optical absorption then increases up to 50%. This theoretical prediction was subsequently confirmed in 2019 by Nematpour *et al.*, who experimentally constructed a mirrored and repeated structure using silicon and silicon oxide with graphene sandwiched inside [29]. Furthermore, by introducing non-mirrored repeated layers to create asymmetry in the structure, Nematpour *et al.* in 2021 also experimentally demonstrated the enhancement of optical absorption of graphene up to 84% [30].

Despite the abovementioned findings, a question remains regarding how the number of layers in the repeated unit affects the optical absorption of graphene. To address this question, our present work aims to investigate the enhancement of the electric field and the optical absorption of the mirrored structure composed of unit layers made of three and four different dielectric materials, referred to as the ABC and ABCD structures, respectively. The transfer matrix method is employed, in section 2, to describe how light propagates through each layer of the structure using matrix formalism related to light propagation through individual layers and pairs of different layers. This approach enables the construction of a transfer matrix that relates the electric field at two distinct positions. After deriving the transfer matrix, electric field enhancement and optical absorption are analytically and numerically calculated for each structure. We meticulously explore scenarios with and without graphene at the centers of different stack configurations (ABC and ABCD). Our results, presented and discussed in section 3, reveal that the ABC and ABCD structures exhibit distinct characteristics for fine-tuning optical absorption. Section 4 concludes by offering insight into the potential future direction of the study.

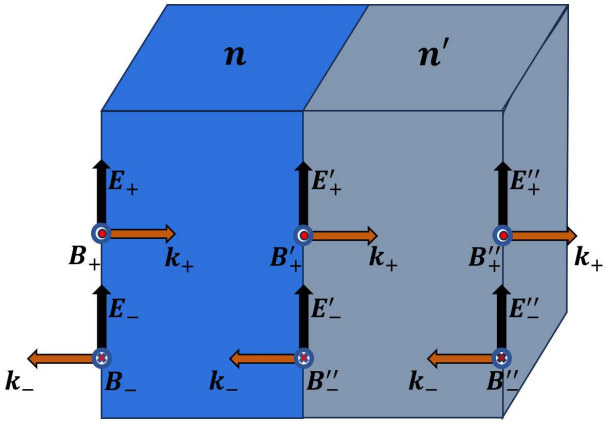

Figure 1. Illustration of light propagation between two media with refractive indices n and n'. Arrows directed to the right (left) sides denote the incident (reflected) light. At the interface of the two media, the electric fields of the incident light and reflected light point in the same direction. Consequently, the magnetic fields of the incident light are opposite to those of the reflected light.

2. Transfer Matrix Method

Let us consider a schematic of light propagation in figure 1, in which the light transverses through media with refractive indices n and n'. In this context, we denote $E^{(+)}$ and $E^{(-)}$ as the electric fields of the right- and left-going light at the initiation of the propagation in medium n. The $E''^{(+)}$ and $E''^{(-)}$ terms are the electric fields of the right and left-going light at the

end of the propagation in medium n'. The description of light propagation is encapsulated by the matrix equations known as the transfer matrix method [31,32]. This is a simple and accurate method for describing wave propagation inside several connected media and has been frequently applied in photonics and quantum transport research fields [18, 31–33]. Within this method, $E^{(+)}$ and $E^{(-)}$ are expressed in terms of $E''^{(+)}$ and $E''^{(-)}$. Consequently, we obtain the following matrix equation,

$$\begin{bmatrix} E^{(+)} \\ E^{(-)} \end{bmatrix} = M_{0|n} P_n M_{n|n'} P_{n'} \begin{bmatrix} E''^{(+)} \\ E''^{(-)} \end{bmatrix}, \tag{1}$$

where P_n represents the propagation matrix of medium n, while $M_{n|n'}$ is the matching matrix between medium n and n'. The product of all P_n and $M_{n|n'}$ is commonly referred to as the transfer matrix of the structure. The two matrices, P_n and $M_{n|n'}$, are given as follows:

$$M = \frac{1}{2} \begin{bmatrix} \left(1 + \frac{n'}{n}\right) & \left(1 - \frac{n'}{n}\right) \\ \left(1 - \frac{n'}{n}\right) & \left(1 + \frac{n'}{n}\right) \end{bmatrix}, \tag{2}$$

$$P_n = \begin{bmatrix} e^{-ikl} & 0 \\ 0 & e^{ikl} \end{bmatrix}, \tag{3}$$

where i is an imaginary number and $k = 2n\pi/\lambda$ represents the wavevector of light with a wavelength of λ. Here, we configure each layer's thickness to be a quarter of the wavelength to ensure there are no reflections when the light propagates through the layer [33]. Therefore, P_n can be simplified to:

$$P_n = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix} \equiv P, \tag{4}$$

where this simplification is independent of the thickness and the refractive index.

We then extend our calculation to the case of the multilayer structures, as depicted in figure 2. In figures 2(a) and 2(b), we illustrate the structures composed of repeated unit layers ABC and ABCD, respectively. The dielectric materials in the unit layers can possess refractive indexes different from each other. The repeated unit layers are mirrored at the center, hence referred to as a mirrored structure. We use r to denote the number of repetitions of the unit layers before mirroring. Generally speaking, the light propagation from the air through the mirrored structure can be defined as the product of the transfer matrices of the unit layers. The transfer matrices of each structure consist of the following matrix

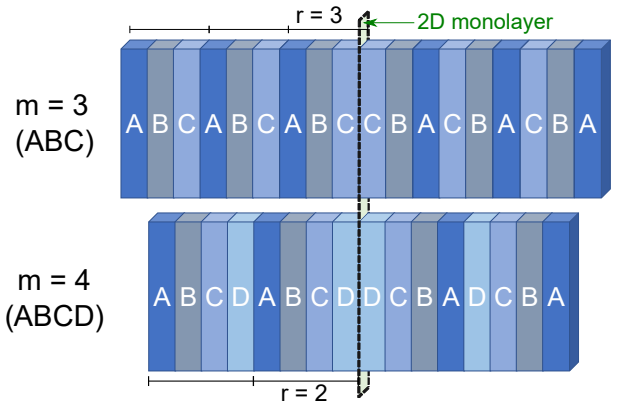

Figure 2. Examples of one-dimensional multilayer dielectric stacks with mirror symmetry. The top image depicts three dielectrics ($m = 3$) in the unit layers and repetition number $r = 3$. The bottom one depicts four dielectrics ($m = 4$) in the unit layer and repetition number $r = 2$.

equations:

$$J_{1(m)} = M_{0|1} P \ldots M_{m-1|m} P, \tag{5a}$$
$$J_{2(m)} = M_{m|1} P \ldots M_{m-1|m} P, \tag{5b}$$
$$J_{3(m)} = P M_{m|m-1} \ldots P M_{1|m}, \tag{5c}$$
$$J_{4(m)} = P M_{m|m-1} \ldots P M_{1|0}, \tag{5d}$$

where the index $x = 1, 2, 3, 4$ represents the extent to which media is used in the unit layers, the index 0 denotes the air, and $m = 3$ ($m = 4$) for the ABC (ABCD) structure. The notations of "..." in equation (5a) represent the multiplication of several matrices M and P (or P and M) consecutively according to the index values in the equation. For example, J_1 for the ABC structure is the result of the sequential multiplication of matrices $M_{0|1}, P, M_{1|2}, P, M_{2|3}, P$, i.e., $J_{1(3)} = M_{0|1} P M_{1|2} P M_{2|3} P$. The same principle also applies to the other J matrices.

Now consider the structure in figure 2(a), where the unit layers consist of ABC layers and the refractive index ratio of the system is defined as $\alpha_3 = n_A n_C / n_B$. The transfer matrices defined in equation (5) for this unit layer are given as follows:

$$J_{1(3)} = \frac{i}{2} \begin{bmatrix} \frac{n_C}{\alpha_3} + \alpha_3 & -\frac{n_C}{\alpha_3} + \alpha_3 \\ \frac{n_C}{\alpha_3} - \alpha_3 & -\frac{n_C}{\alpha_3} - \alpha_3 \end{bmatrix}, \tag{6}$$

$$J_{2(3)} = \frac{i}{2} \begin{bmatrix} \frac{n_C}{\alpha_3} + \frac{\alpha_3}{n_C} & -\frac{n_C}{\alpha_3} + \frac{\alpha_3}{n_C} \\ \frac{n_C}{\alpha_3} - \frac{\alpha_3}{n_C} & -\frac{n_C}{\alpha_3} - \frac{\alpha_3}{n_C} \end{bmatrix}, \tag{7}$$

$$J_{3(3)} = \frac{i}{2} \begin{bmatrix} \frac{\alpha_3}{n_C} + \frac{n_C}{\alpha_3} & \frac{\alpha_3}{n_C} - \frac{n_C}{\alpha_3} \\ -\frac{\alpha_3}{n_C} + \frac{n_C}{\alpha_3} & -\frac{\alpha_3}{n_C} - \frac{n_C}{\alpha_3} \end{bmatrix}, \tag{8}$$

$$J_{4(3)} = \frac{i}{2}\begin{bmatrix} \frac{\alpha_3}{n_C}+\frac{1}{\alpha_3} & \frac{\alpha_3}{n_C}-\frac{1}{\alpha_3} \\ -\frac{\alpha_3}{n_C}+\frac{1}{\alpha_3} & -\frac{\alpha_3}{n_C}-\frac{1}{\alpha_3} \end{bmatrix}. \quad (9)$$

Hence, the light propagation through such a structure consisting of repeating ABC layers is represented by the following matrix equation,

$$\begin{bmatrix} E_0^{(+)} \\ E_0^{(-)} \end{bmatrix} = J_{1(3)} J_{2(3)}^{r-1} M_C J_{3(3)}^{r-1} J_{4(3)} \begin{bmatrix} E_{N+1}^{(+)} \\ 0 \end{bmatrix}, \quad (10)$$

where $E_0^{(+)}$, $E_0^{(-)}$ and $E_{N+1}^{(+)}$ are the incident, reflected, and transmitted electric fields of the structure, respectively, while M_C is the matching matrix for the material at the center. If there is no material at the center of the multilayer structure, M_C is an identity matrix.

Figure 2(b) illustrates the structure of the unit layers composed of ABCD layers. The system's refractive index ratio defined as $\alpha_4 = n_A n_C/(n_B n_D)$, where the transfer matrices are as follows:

$$J_{1(4)} = \frac{1}{2}\begin{bmatrix} \frac{1}{\alpha_4}+\alpha_4 n_D & \frac{1}{\alpha_4}-\alpha_4 n_D \\ \frac{1}{\alpha_4}-\alpha_4 n_D & \frac{1}{\alpha_4}+\alpha_4 n_D \end{bmatrix}, \quad (11)$$

$$J_{2(4)} = \frac{1}{2}\begin{bmatrix} \frac{1}{\alpha_4}+\alpha_4 & \frac{1}{\alpha_4}-\alpha_4 \\ \frac{1}{\alpha_4}-\alpha_4 & \frac{1}{\alpha_4}+\alpha_4 \end{bmatrix}, \quad (12)$$

$$J_{3(4)} = \frac{1}{2}\begin{bmatrix} \frac{1}{\alpha_4}+\alpha_4 & -\frac{1}{\alpha_4}+\alpha_4 \\ -\frac{1}{\alpha_4}+\alpha_4 & \frac{1}{\alpha_4}+\alpha_4 \end{bmatrix}, \quad (13)$$

$$J_{4(4)} = \frac{1}{2}\begin{bmatrix} \frac{1}{\alpha_4 n_D}+\alpha_4 & -\frac{1}{\alpha_4 n_D}+\alpha_4 \\ -\frac{1}{\alpha_4 n_D}+\alpha_4 & \frac{1}{\alpha_4 n_D}+\alpha_4 \end{bmatrix}. \quad (14)$$

Thus, the propagation of light through the stacks can be represented by the following matrix equation,

$$\begin{bmatrix} E_0^{(+)} \\ E_0^{(-)} \end{bmatrix} = J_{1(4)} J_{2(4)}^{r-1} M_C J_{3(4)}^{r-1} J_{4(4)} \begin{bmatrix} E_{N+1}^{(+)} \\ 0 \end{bmatrix}, \quad (15)$$

where $E_0^{(+)}$, $E_0^{(-)}$ and $E_{N+1}^{(+)}$ are defined similarly to the case of the structure with repeating ABC layers.

Using the transfer matrix method, we investigate the behavior of the electric fields inside these structures, with a specific focus on examining the optical absorption of the structures when an undoped graphene layer is placed at the center. Previous studies have employed the transfer matrix method for monolayer graphene [18, 34, 35], in which graphene is treated as a conducting thin layer with a negligible thickness when compared to the wavelength of light. In the presence of graphene, the propagation of light is described by

$$\begin{bmatrix} E_0^{(+)} \\ E_0^{(-)} \end{bmatrix} = J_{1(3,4)} J_{2(3,4)}^{r-1} M_G J_{3(3,4)}^{r-1} J_{4(3,4)} \begin{bmatrix} E_{N+1}^{(+)} \\ 0 \end{bmatrix}. \quad (16)$$

where M_G represents the matching matrix of the graphene layer, i.e.,

$$M_G = \frac{1}{2}\begin{bmatrix} 2+\frac{Z_0}{n_m}\sigma & \frac{Z_0}{n_m}\sigma \\ -\frac{Z_0}{n_m}\sigma & 2-\frac{Z_0}{n_m}\sigma \end{bmatrix}, \quad (17)$$

where $Z_0 = \sqrt{\mu_0/\epsilon_0} = 377\Omega$ is the impedance of the vacuum and $\sigma = e^2/4\hbar$ is the optical conductivity of undoped graphene [15]. The refractive index n in equation (17) is determined by the material surrounding graphene, which is symbolized by the m index. Specifically, $n = n_C$ and $n = n_D$ for the cases of figures 2(a) and 2(b), respectively. From equations (10), (15), and (16), we can derive the relation between the transmitted and reflected electric fields, $E_{N+1}^{(+)}$ and $E_0^{(-)}$, concerning the incident electric field $E_0^{(+)}$. Subsequently, the electric field at the center of the structure can be calculated after establishing the relation between the incident, transmitted, and reflected electric fields. In the case of no graphene at the center of the stacks, the electric field at the center, $E_c = E_c^{(+)} + E_c^{(-)}$, is determined by

$$\begin{bmatrix} E_c^{(+)} \\ E_c^{(-)} \end{bmatrix} = \left(J_{1(3,4)} J_{2(3,4)}^{r-1}\right)^{-1} \begin{bmatrix} E_0^{(+)} \\ E_0^{(-)} \end{bmatrix}. \quad (18)$$

On the other hand, in the presence of graphene, the electric field at the center of the structure is defined by

$$\begin{bmatrix} E_c^{(+)} \\ E_c^{(-)} \end{bmatrix} = \left(J_{1(3,4)} J_{2(3,4)}^{r-1} M_G\right)^{-1} \begin{bmatrix} E_0^{(+)} \\ E_0^{(-)} \end{bmatrix}. \quad (19)$$

Using equation (19) we can calculate the absorption probability $A = 1 - T - R$ where the transmission T and reflection probabilities R can be obtained from equation (16) as $T = |E_{N+1}^{(+)}/E_0^{(+)}|^2$ and $R = |E_0^{(-)}/E_0^{(+)}|^2$, respectively. This procedure will be more detailed in section 3.2. Note that, for easy reproducibility, we open the detailed calculations using the methods described in this section in a public repository [36].

3. Results and discussion

In this section, we present the electric field at the center (E_c) of each structure shown in figure 2. We begin by

displaying E_c as a function of the repetition number r for the case without graphene. Then, graphene is introduced into the structure and we calculate E_c, where in this context E_c corresponds to the electric field on the surface of graphene. Finally, the optical absorption of the structure as a function of both r and the refractive index is discussed.

3.1. Electric field enhancement in multilayer dielectric stacks (without graphene)

The value of E_c is obtained from equation (18). The reflected electric field $E_0^{(-)}$ is calculated by solving equation (15). It is important to note that Liu *et al.* have shown that the $E_0^{(-)} = 0$ for the mirrored structure, owing to the symmetry of the structure [28, 33]. Furthermore, the repetition of the unit layers is represented by the $(r-1)$th power of the matrices J_2 and J_3, as defined in equations (7) and (12).

Let us begin with the case of repeating the ABC structure shown in figure 2(a). The matrices $J_{2(3)}$ and $J_{3(3)}$, as defined in Eqs. (7) and (8), become an identity matrix when they are multiplied by themselves. Since any matrix remains unchanged when multiplied with an identity matrix, the equation transforms into a repeating pattern that relies on the number of layers employed in the structure. This gives only two possibilities for E_c as indicated below:

$$\frac{E_c}{E_0^{(+)}} = \frac{1}{\alpha_3}, \quad r = 1, 3, 5, \ldots, \tag{20}$$

and

$$\frac{E_c}{E_0^{(+)}} = 1, r = 2, 4, 6, \ldots. \tag{21}$$

Figure 3(a) shows the enhancement of the electric field at the center of the structure, denoted as $E_c/E_0^{(+)}$, as a function repetition number r obtained through the numerical calculation. We also compare these numerical results with the analytical formulas presented in equations (20) and (21). The value of n_A, n_B, n_C, and n_D are $2, 1, 2$, and 1, respectively. The $E_c/E_0^{(+)}$ ratio can be seen to exhibit only two distinct values as given by equations (20) and (21). For odd r, the $E_c/E_0^{(+)}$ is less than unity and depends on the refractive index. However, selecting $n_B > n_A n_C$ allows us to get an enhanced electric field. On the other hand, for even r, the $E_c/E_0^{(+)}$ ratio remains consistently equal to unity. This significant enhancement implies that the velocity of light within the formation is considerably reduced due to the energy conservation principle, a phenomenon commonly observed in fractal structures [37].

Next, we consider the case of repeating the ABCD structure as shown in figure 2(a). In this

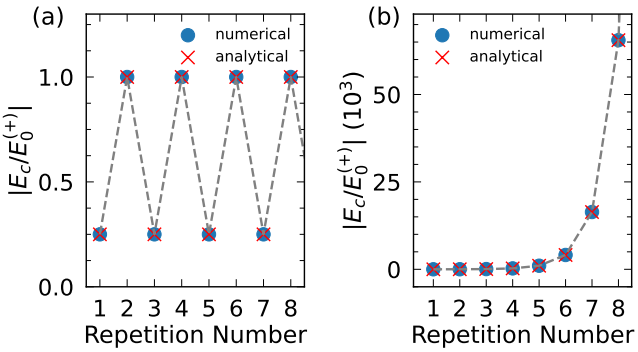

Figure 3. Electric field enhancement at the center of multilayer dielectric stack without graphene for (a) three dielectrics (ABC structure) in the unit layer and (b) four dielectrics (ABCD structure) in the unit layer. The enhancement is plotted as a function of unit layers repetition r. The refractive indices n_A, n_B, n_C, and n_D are set to 2, 1, 2, and 1, respectively, so that $\alpha_3 = \alpha_4 = 4$.

structure, equation (12) does not transform into an identity matrix when it is multiplied by itself, so we straightforwardly multiply all the matrices involved in equation (18). The following analytical formula for $E_c/E_0^{(+)}$ is derived:

$$\frac{E_c}{E_0^{(+)}} = \alpha_4^r. \tag{22}$$

Figure 3(b) shows the enhancement of the electric field at the center of the structure, $E_c/E_0^{(+)}$, as a function repetition number r through the numerical calculation and the analytical formula in equation (22). As evident from the figure, the $E_c/E_0^{(+)}$ ratio increases monotonically with increasing r. However, it is noteworthy that the enhanced electric field is achieved only if $n_A n_C > n_B n_D$. Otherwise, the electric field diminishes with increasing r.

3.2. Optical absorption in graphene sandwiched inside multilayer dielectric stacks

In the presence of graphene, the propagation of light through the structure is described by equation (16). The optical absorption probability A of the structure is determined by

$$A = 1 - T - R, \tag{23}$$

where the transmission T and reflection probabilities R obtained from equation (16) are defined as $T = |E_{N+1}^{(+)}/E_0^{(+)}|^2$ and $R = |E_0^{(-)}/E_0^{(+)}|^2$, respectively. It is worth noting that the presence of graphene disrupts the symmetry of the structure, leading to a non-zero $E_0^{(-)}$. As discussed in section 3.1, there are two possible scenarios for A in the case of repeated ABC

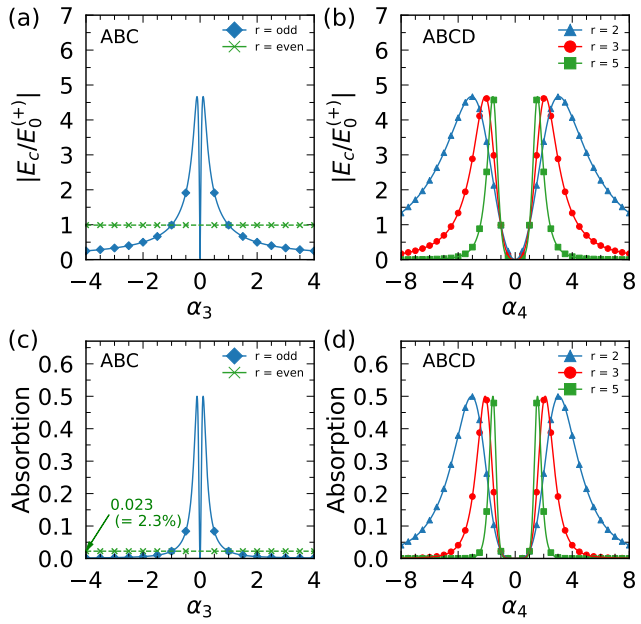

Figure 4. (a,b) Electric field enhancement and (c,d) light absorption as a function of refractive index ratio α for (a, c) ABC and (b, c) ABCD structures. The symbols denote data points obtained by numerically solving equations (19) and (23) with our Python codes, while the lines represent the results from analytical formulas [equations (24)–(26) for the absorption and (27)–(29) for the electric field enhancement.

structure with graphene at the center, depending on r as follows:

$$A = \frac{4Z_0\sigma\alpha_3^2}{\left(2\alpha_3^2 + Z_0\sigma\right)^2}, \quad r = 1, 3, 5, \ldots, \tag{24}$$

and

$$A = \frac{4Z_0\sigma}{\left(2 + Z_0\sigma\right)^2}, \quad r = 2, 4, 6, \ldots. \tag{25}$$

On the other hand, in the case of repeated ABCD structure with graphene at the center, we obtain only one formula for any r:

$$A = \frac{4Z_0\sigma(\alpha_4)^{2r}}{\left(2 + (\alpha_4)^{2r} Z_0\sigma\right)^2}. \tag{26}$$

To analyze the absorption probabilities obtained in equations (24)–(26), we also calculate the magnitude of the electric field E_c on the surface of graphene. The E_c field is calculated from equation (19) in conjunction with equation (16). In the case of repeated ABC structure with graphene at the center, there are two possibilities of E_c depending on r as follows:

$$\frac{E_c}{E_0^{(+)}} = \frac{2\alpha_3}{2\alpha_3^2 + Z_0\sigma}, \quad r = 1, 3, 5, \ldots, \tag{27}$$

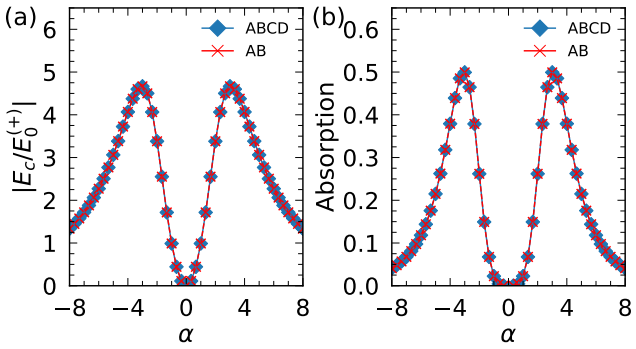

Figure 5. Comparison of (a) electric field enhancement, and (b) optical absorption of graphene in the AB and ABCD structures, adjusted so both the structures can have the same refractive index ratio per layer despite having a different number of materials used. The symbols denote numerical data points, while the lines represent the analytical results.

and

$$\frac{E_c}{E_0^{(+)}} = \frac{2 + 2Z_0\sigma}{2 + Z_0\sigma}, \quad r = 2, 4, 6, \ldots. \tag{28}$$

For the ABCD structure with graphene at the center, we obtain only one formula of the electric field enhancement for any r as follows:

$$\frac{E_c}{E_0^{(+)}} = \frac{2\alpha_4^r}{2 + Z_0\sigma(\alpha_4)^{2r}} \tag{29}$$

Figure 4 shows the electric field enhancement $E_c/E_0^{(+)}$ and absorption A as a function of the corresponding refractive index ratio (α_3 or α_4) for each structure. For optimal results, it is essential to consider this ratio, rather than focusing on the individual refractive indices of the materials. As we can see, the peak of $E_c/E_0^{(+)}$ corresponds to the peak of A, and we always obtain a maximum value of A

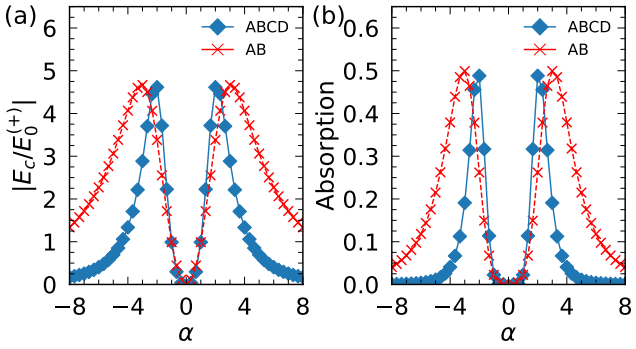

Figure 6. (a) Electric field enhancement and (b) light absorption of graphene in the AB and ABCD structures, adjusted so the ABCD structure used more multilayer units than the AB structure. The symbols denote numerical data points, while the lines represent the analytical results.

of 50% at a certain value of α, except in the case of ABC structure with even r. This correspondence arises from the general definition of optical absorption, which is essentially the Joule heat generated on the graphene layer. The Joule heat is proportional to the square of the electric field, explaining the same peak position between A and $E_c/E_0^{(+)}$. Notably, when comparing equations (24)–(26) with equations (27)–(29), we understand that $A \propto (E_c/E_0^{(+)})^2$. For the ABC structure, as depicted in figure 4(a), the odd r gives nearly five times enhancement of the electric field at α_3 close to zero, while the even r provides no enhancement. The peak of $E_c/E_0^{(+)}$ for odd r corresponds to the 50% absorption (i.e., $A = 0.5$) as shown in figure 4(c), while the even r leads to the conventional 2.3% absorption (i.e., $A = 0.023$) in undoped graphene. However, the A value rapidly decreases with increasing α for odd r, and it remains constant for even r. These results are not trivial and we would not expect them unless we do such calculations as in our present work. The difference in the electric field enhancement and optical absorption results for the odd and even r stems from the interference phenomena within the layered structure. The main reason why odd r gives larger electric field enhancement (hence optical absorption) than even r is that the electric fields on graphene in the case of odd r interfere with each other more constructively than in the case of even r.

For the repeated ABCD structure, we also achieve the maximum of five times enhancement of the electric field, regardless of any r, which corresponds to 50% absorption, as shown by figures 4(b) and 4(d). The peak position of A in α depends on r, and increasing r shifts the peak to lower α. Thus, it is possible to obtain 50% absorption even without a large number of layers, provided that α is large. It is worth noting that for the ABCD structure, we can obtain a similar result to that obtained by Nulli *et al.* [28] when we adjust the four ABCD materials to mimic the structure made of two AB materials, resulting in the same electric field enhancement and absorption as shown in figures 5(a)–(b). However, in general, the ABCD structure depicts a more complex pattern compared to the AB structure when we *do not* set C = A and D = B. Mathematically, the equivalence occurring when C = A and D = B is due to the interference within both structures exhibiting the same pattern. Although the maximum value of the enhancement is the same as previous works (50%), our present study has further novelty in the sense that we can obtain the maximum absorption with a smaller repetition number than, for example, the work of Nulli *et al.* [28], who considered only the AB structure. We can also fine-tune other absorption values with more options with the variation of the structure of unit layers.

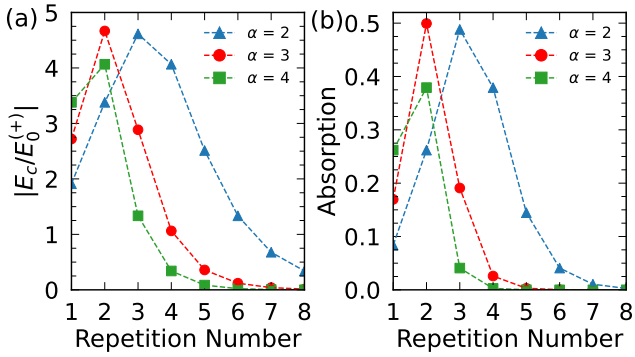

Figure 7. (a) Electric field enhancement and (b) light absorption of graphene in the ABCD structures with different refractive index ratios plotted as a function of the number of multilayer repetitions r.

Furthermore, we can adjust the number of multilayer units used within the structure to pinpoint the exact refractive index ratio needed to reach the maximum electric field and light absorption, as demonstrated in figures 6(a)–(b). Finally, we can determine the specific number of multilayer sets required to maximize both the electric field enhancement and light absorption values, depending on the refractive index ratio of the multilayer set, as shown in figures 7(a)–(b). Considering these findings, we can deduce that there is a distinct pattern to both electric field enhancement and light absorption rate for a two-dimensional layer in the middle of the structure, depending on how many kinds of materials are used to construct the structure. Therefore, by utilizing these properties we can find several applications such as solid-state lighting [38], Fano resonators [39], integrated circuits [40], photodetectors [41], photovoltaics [42], waveguiding structures [43], and other optoelectronic applications. However, there are some limitations in this study. One of the study's limitations is that we only consider lossless dielectric media. Another limitation is that we only consider simple repetition of the dielectric media. There might be a more complex structure such as using the Fibonacci sequence of dielectric media which might give more exotic optical properties. How to overcome these limitations can be a topic of future study.

3.3. Insight into other 2D materials

There is still a lingering question, especially since we still have not yet applied this multilayer structure to any material other than graphene. What if we were to insert a different, albeit similarly thin material inside the mirrored multilayer structure, such as silicene, which exhibits a more diverse optical conductivity

due to its non-negligible spin-orbit coupling? In general, silicene has a larger optical conductivity value than graphene because of non-negligible spin-orbit interaction and the buckled structure of silicene which induces additional potential between two sublattices [44]. By replacing graphene with silicene, we achieve the same 50% maximum optical absorption but at a different value of refractive index ratio.

4. Conclusion

Utilizing the transfer matrix method, we have formulated explicit equations for graphene's optical absorption at the center of the mirrored dielectric stacks consisting of three or four dielectric materials as the unit layer. Our findings reveal that optical absorption significantly depends on two key factors: the number of repetitions r and the structural composition of the unit layers. When graphene is placed within a unit layer structure comprised of three dielectric materials (the ABC structure) and even values of r, it exhibits an optical absorption of approximately 2.3% within the visible to near-infrared light spectrum, analogous to that of free-standing graphene, showing minimal impact from the surrounding dielectric stacks. In contrast, when we consider the ABC structure with odd values of r or expand to the use of four materials (the ABCD structure) with any values of r, we can achieve remarkably high optical absorption, reaching up to 50%. These enhanced absorption rates are observed at specific refractive indices n for the constituent dielectric materials. This precise control opens up new possibilities for tailoring the optical absorption of graphene, making it exceptionally versatile for a wide range of optoelectronic applications.

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Data availability statement

The data that support the findings of this study are available at: https://github.com/solihinn17/graphene-stack-absorption

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